

a function of both dose (measured by the time of irradiation, t) and depth, we write $P(t,x)$. The linear density of pinning points as a function of dose and depth is

$$n(t,x) = n_0 + P(t,x)/N \quad (5)$$

where n_0 is the number of pinning points per unit length of dislocation line present before irradiation. Presumably the number n_0 represents, primarily, points where three dislocation lines intersect. Since n varies with both t and x , the modulus defect ϕ of an element of volume of the crystal will depend on both t and x . The modulus defect for the whole crystal, $\Phi(t)$, observed at any time t will then be the average of the modulus defect in each layer of the crystal, providing that $\phi \ll 1$ everywhere. Thus

$$\Phi(t) = \frac{1}{b} \int_0^b \phi(t,x) dx \quad (6)$$

where b is the thickness of the crystal in the direction of the incident radiation.

In order to express $\phi(t,x)$ in terms of $n(t,x)$ we may make use of the theory of the decrease in modulus due to dislocation loops devised by Mott and Friedel. Since $n = l^{-1}$, equation (1) becomes

$$\Delta\mu/\mu = N/n^2\xi'$$

which, as discussed in the appendix, may be written as

$$\phi = -\frac{\Delta s_{11}}{s_{11}} = \frac{N}{n^2\xi} \quad (7)$$

where ξ is a slowly-varying function of n whose value is about 2.5. Substituting equations (5) and (7) into (6), we obtain,

$$\Phi(t) = \frac{N}{b\xi} \int_0^b \frac{dx}{(n_0 + P(t,x)/N)^2} \quad (8)$$

if ξ is taken as essentially constant. Correspondingly, Φ_0 , the value of Φ prior to irradiation, is given by

$$\Phi_0 = N/\xi n_0^2. \quad (8a)$$

To carry the calculation further requires the introduction of more detailed assumptions about $P(t,x)$. The following two assumptions will be considered.

Assumption I: The number of pinning points created by the irradiation is assumed to be proportional to the number of F centers created. Now, it is known from optical measurements⁽⁶⁾ that, at all depths in the crystal, the number of F centers per unit volume increases nearly linearly with irradiation time in the range of X-ray dosage required to bring the

modulus increase very close to saturation.* Thus it is possible to introduce the simplification that

$$P(t,x) = f(x) \cdot t \quad (9)$$

where $f(x)$ gives the dependence of the rate of coloring on depth. It was also observed in the optical measurements⁽⁶⁾ that the initial rate of formation of F centers is essentially the same in annealed crystals as in crystals deformed various amounts before irradiation. Consequently, $f(x)$ may be taken to be essentially the same in all the crystals which are being considered here. Equation (8) combined with (8a) then becomes

$$\frac{\Phi(t)}{\Phi_0} = \frac{1}{b} \int_0^b \frac{dx}{(1 + tf(x)/Nn_0)^2} \quad (10)$$

Thus the relative modulus defect, $\Phi(t)/\Phi_0$, depends on irradiation time and on the previous history of the crystal only through variable t/Nn_0 . It is therefore anticipated that curves of Φ/Φ_0 vs. log (irradiation time) for different crystals should differ from each other only by a translation along the log-time axis (i.e. by a change in time scale). As shown in Fig. 5, this prediction is indeed verified within experimental error[†] for crystals subjected to varying amounts of deformation and recovery.

If the constant C is arbitrarily defined as the change in time scale required to bring all of the Φ/Φ_0 vs. log-time curves into coincidence with the curve for crystal No. 6, the quantity C will be proportional to $(Nn_0)^{-1}$. Utilizing this result and equation (8a), we can evaluate quantities proportional to N and n_0 for each crystal in terms of the known quantities C and Φ_0 . Thus,

$$\left. \begin{aligned} N &\approx \Phi_0^{1/3} C^{-2/3} \\ l_0 &= n_0^{-1} \approx (C\Phi_0)^{1/3} \end{aligned} \right\} \quad (11)$$

Values of these quantities are given in Table 4 under the heading "Assumption I" for a number of crystals having different mechanical histories.

Assumption II: The fact that optical measurements show an initial rate of formation of F centers which is essentially independent of the state of deformation or anneal of the specimen implies that F centers are not formed at dislocations during irradiation[‡]. The

* In the optical measurements (reference 6) direct irradiation was used, in contrast to the present experiments in which the specimen was translated in front of the X-ray beam. Consequently a 10-min irradiation in the present experiments is equivalent to only 2 min in the optical experiments.

† It should be noted that the experimental error in determining $\Phi(t)$ is largest when $\Phi(t)$ approaches zero, because of the difficulty in determining the exact saturation value of the modulus.

‡ The interpretation of these optical data is presented in greater detail elsewhere.⁽⁶⁾

modulus increase resulting from X-irradiation is supposed to arise, however, from the pinning of dislocations by the action of the absorbed radiation. It is therefore conceivable that the modulus increase is due to *direct* interaction between the products of the irradiation (excitons and/or free electrons and holes) and dislocations, even though *F* centers are not primarily formed in this way. If this were the case, the density of pinning points *P* created by the irradiation would not be proportional to the number of *F* centers (as in Assumption I) but rather to the density of dislocation lines *N*. We may therefore state, for Assumption II, that equation (9) still applies, except that *f(x)* is no longer independent of the histories of the various specimens, but rather that it is proportional to *N*. The result obtained under Assumption I, that the curves of $\Phi(t)/\Phi_0$ vs. log (irradiation time) for crystals given different prior treatments are displaced from each other by translation along the log-time axis, is therefore still valid under Assumption II. However, the relative shift *C* of the curves is now proportional to n_0^{-1} and independent of *N* (since the proportionality of *f(x)* to *N* cancels this quantity out of equation 10). Under these conditions, we obtain for Assumption II, in place of equations (11),

$$\left. \begin{aligned} N &\approx \Phi_0 C^{-2} \\ l_0 &= n_0^{-1} \approx C \end{aligned} \right\} \quad (12)$$

Values of the quantity $\Phi_0 C^{-2}$ for various crystals are given in Table 4, under the heading "Assumption II," while the quantity *C* itself is already listed in an earlier column.

A choice between the two assumptions just discussed may be made on the basis of a comparison of numbers proportional to *N* and l_0 calculated in terms of these assumptions. For example, when the quantities $\Phi_0^{1/3} C^{-2/3}$ for different crystals are compared, one finds that interpretation of the data in terms of Assumption I leads to the conclusion that *N* increases roughly in proportion to the amount of deformation and decreases somewhat as recovery is

allowed to proceed. The values of $(C\Phi_0)^{1/3}$ show that l_0 is very nearly the same in all cases except one, viz. that of crystal No. 10. This crystal was evidently either deformed at a substantially different rate or else was very different in its history prior to deformation. Its large decrement (15×10^{-3} as against a value of 4×10^{-3} for crystal No. 6) as well as its large Φ_0 -value are consistent with the large value of l_0 .

On the other hand, interpretation of the data in terms of Assumption II leads to numerical values which are difficult to reconcile with the prior treatments received by the various crystals. A comparison of $\Phi_0 C^{-2}$ for crystals Nos. 5 and 9 shows that *N* is required to decrease by a factor greater than 10 during recovery from 180 min to 5760 min for specimens deformed very nearly the same amount. It is difficult to reconcile such a large decrease in *N* as the result of this recovery with the fact that such recovery does not greatly affect other properties of the material (such as X-ray line broadening) which are sensitive to dislocation density. The large difference in values of $\Phi_0 C^{-2}$ for crystals No. 6 and 10 is also difficult to explain, since the deformations and recovery times of these two crystals are very nearly the same. Finally, according to Assumption II, *N* would vary much faster than linearly with the amount of deformation. This result also does not seem reasonable. In general, then, it is seen that Assumption I leads to the most satisfactory agreement between the calculated numbers proportional to *N* and l_0 and the histories of the specimens.

Evaluation of the integral in equation (10) requires knowledge of the form of *f(x)*. In terms of Assumption I, *f(x)* should have the same form as the variation in the density of *F* centers with depth below the irradiated surface. This variation has been determined experimentally and found to be roughly proportional to the rate of absorption of X-ray energy at each depth in the crystal.⁽¹⁸⁾ The functional form of this depth dependence is rather complex, but under the conditions of irradiation used in the present experiments,

TABLE 4. Analysis of the modulus changes in cold-worked NaCl crystals

Crystal No.	Def. %	Room temperature recovery (min)	$\Phi_0 \times 10^2$	<i>C</i>	Assumption I		Assumption II	$N\beta^{2/3}$ ($\text{cm}^{-2} \times 10^{-8}$)	$l_0\beta^{-1/3}$ ($\text{cm} \times 10^6$)
					$\Phi_0^{1/3} C^{-2/3}$	$(C\Phi_0)^{1/3}$	$\Phi_0/C^2 \times 10^3$		
6	4.20	1700	1.68	1.00	0.258	0.256	16.8	32.0	3.61
10	4.08	1440	5.02	2.47	0.201	0.505	8.22	25.0	7.04
5	2.14	180	1.01	1.65	0.154	0.255	3.68	19.2	3.61
9	2.00	5760	0.495	3.79	0.068	0.265	0.343	8.5	3.75
4	(as received)		0.413	3.79	0.067	0.250	0.287	8.3	3.53